HILGARDIA

A JOURNAL OF AGRICULTURAL SCIENCE

PUBLISHED BY THE

CALIFORNIA AGRICULTURAL EXPERIMENT STATION

VOL. 1 MAY, 1925 NO. 4

AN ACCURATE METHOD OF CALCULATING ICE CREAM MIXES

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INTRODUCTION

One of the most important factors in the manufacture of ice cream of high quality is the proper proportioning of the ingredients. Formulas are used by a few ice cream manufacturers, but a thorough knowledge of how to standardize the mix (mixture) by using ingredients with varying composition, and thereby bringing about the desired per cent of fat, milk-solids-not-fat, total solids, *et cetera*, is of greater value than the possession of a great number of formulas.

To be able to calculate a mix simply and accurately is very useful to the ice cream manufacturer. Many requests have been received by the Division of Dairy Industry, asking how this can be done. Several methods have been published during the past year or so for computing mixes, but the one presented here seems simpler than any other yet suggested. This method makes use of algebraic formulas, and is more accurate than the "cut and try" method used by many. Rules based on these formulas are given for those who would prefer rules to algebraic formulas.

Composition of Ingredients: Before the composition of a mix can be calculated, a standard must be decided upon. By a standard is meant the per cent of milk-solids-not-fat, sugar, gelatin, and water for gelatin, that the finished mix is to contain. Every ice cream manufacturer has some kind of standard. The sum of the per cents of solids in the ingredients used determines the per cent of total solids in the mix.

The California State law requires 10% fat in plain ice cream. An excellent ice cream can be made by using the following standard, which is the one now used by the University of California:

Fat, 10.5%
Milk-solids-not-fat, 10%
Sugar, 14.5%
Gelatin, .5%
Water, 3% for hydrating gelatin.

Before any standard can be used, its ingredients must, of course, be known. *It is convenient to have the ingredients tabulated as in table 1.*

Ingredients:

TABLE 1

LIST OF INGREDIENTS FOR REFERENCE AND KEY TO CASES

1. Cream
2. Milk—whole
3. Milk—whole condensed (evaporated milk)
4. Milk—whole condensed and sweetened
5. Skim milk
6. Skim milk condensed
7. *Skim milk condensed and sweetened*
8. Whole milk powder
9. Skim milk powder
10. Butter
11. Sugar } all cases
12. Gelatin } all cases
13. Gelatin water } all cases
14. Water

Case I. Ingredients—Cream, milk, condensed skim milk
Case II. Ingredients—*1, 2, 6+3 or 4 or 6
Case III. Ingredients—1, 2, 3
Case IV. Ingredients—1, 2, 4
Case V. Ingredients—1, 5
Case VI. Ingredients—1, 2+condensation
Case VII. Ingredients—1, 5, 6
Case VIII. Ingredients—Left over, 1, 2
Case IX. Ingredients—10, 1, 6
Case X. Given amounts of two or more ingredients
Case XI. *Other standards*

Cream: Cream is used quite extensively as the main source of fat in ice cream. The more fat it contains, the less of other solids it can have. The fat crowds out the milk-solids-not-fat as shown by the following table:

* These numbers refer to the numbers given to the ingredients listed in Table 1.

Milk: Legal milk must contain at least 3 per cent of fat. It must be remembered that milk increases the milk-solids-not-fat about twice as fast as it does the fat. It is almost universally used in building up fat and milk-solids-not-fat in ice cream.

Sweetened Condensed Milk: The constituents of sweetened condensed milk are the same as those of fresh milk, except that they have been concentrated by evaporating in vacuo and have had a varying amount of sucrose, usually about 42%, added to them. This product is used to a considerable extent in the manufacture of ice cream.

Evaporated Milk: Evaporated milk is practically the same product as sweetened condensed milk except that it lacks the addition of the sucrose.

Condensed Skim Milk: It is necessary to build up the milk-solids-not-fat by means of some condensed product; this is usually done with condensed skim. The per cent of solids in the condensed skim milk varies in different plants and varies from day to day in the same plant. Therefore, a very careful check should be kept on the per cent of milk solids in the condensed skim. It usually is the cheapest source of additional milk-solids-not-fat. Probably the most desirable per cent of milk-solids-not-fat in condensed skim is 32.

Skim Milk Powder: Skim milk powder is also used to some extent in localities where it is difficult to secure the condensed skim. The powder makes a very good quality of ice cream, and while it seems perfectly dry to the hand, it contains from two to three per cent moisture. For all practical purposes, it can be considered as containing 97 per cent milk-solids-not-fat.

Whole Milk Powder: Whole milk powder can be purchased from powder plants, but must be used within a short time after manufacture. It is made by powdering the whole milk, and may be used where conditions and price will warrant.

Butter: Sweet butter is used to some extent in the manufacture of ice cream. Sweet butter is that which contains no salt. Since its composition varies, it should be purchased on a fat basis. It usually contains approximately 83 per cent fat. A first class ice cream can be made from it. Even salted butter may be used by melting it at a temperature not exceeding 150° F. and allowing the salt, curd and water to settle out. After they have settled out, they should be drawn off and if necessary, the oil should be washed with warm water. By this process a butter oil practically 100 per cent pure may be secured.

Granulated Sugar: Granulated sugar, either beet or cane, may be used in the manufacture of ice cream, and should be figured as containing 100 per cent solids. Washburn says "that sugar, if stored in a damp place, will absorb only ¼% moisture or less."

Corn Sugar: Corn sugar contains about 10 per cent moisture, is only about 60 per cent as sweet as cane sugar, and may be used to a limited extent to replace granulated sugar.

Gelatin: Gelatin is a very important ingredient in the manufacture of ice cream, but is used only in small amounts, about .5 of one per cent or less. It absorbs moisture very readily. The best grades of gelatin contain from 5 to 10 per cent moisture and none but the best should be used.

Gelatin Water: Some ice cream manufacturers dissolve the gelatin in water at the rate of six parts of water to one of gelatin. It should be dissolved in cold water and heated in a steam jacketed kettle from 165° to 185° F., the temperature varying in different plants. The gelatin solution is added to the mix while it is being held at a pasteurizing temperature.

The Method of Computing the Mix: As suggested above, the method here explained involves the use of algebraic formulas and rules based on these formulas. Thus, there is no "cutting and trying" in it, and the results obtained, when carefully "checked," are bound to be correct—not just rough approximations. For those who know a little elementary algebra, the formulas will be found very simple; a little practice with them will insure both speed and accuracy in computing mixes. Those who have no knowledge whatever of algebra will find it best to use the *rules* at first, but even *they* should compare the rules with the formulas and should study the examples worked out by means of the latter. If they will do this, it is believed that they will soon be able also to use the formulas efficiently and will prefer them to the more cumbersome rules.

Although some examples are worked out showing how to adapt the formulas to other standards, the standard used in this circular is the following:

Fat, 10.5%
Milk-solids-not-fat, 10%
Sugar, 14.5%
Gelatin, .5%
Water, 3% for hydrating gelatin.

Skim Milk: Skim milk may be used in increasing satisfactorily the milk-solids-not-fat.

*TABLE 2

SHOWING PER CENT OF FAT, AND MILK-SOLIDS-NOT-FAT IN MILK AND CREAM

Product	Per cent fat	Per cent milk-solids-not-fat	Product	Per cent fat	Per cent milk-solids-not-fat
Water	0.00	0.00	Cream	30.0	6.24
Skim milk	.02	8.91	Cream	31.0	6.15
Milk	3.0	8.61	Cream	32.0	6.06
Milk	3.5	8.71	Cream	33.0	5.97
Milk	4.0	8.81	Cream	34.0	5.88
Milk	4.5	8.92	Cream	35.0	5.79
Milk	5.0	9.02	Cream	36.0	5.70
Cream	15.0	7.57	Cream	37.0	5.61
Cream	16.0	7.48	Cream	38.0	5.52
Cream	17.0	7.40	Cream	39.0	5.44
Cream	18.0	7.31	Cream	40.0	5.35
Cream	19.0	7.22	Cream	41.0	5.26
Cream	20.0	7.13	Cream	42.0	5.17
Cream	21.0	7.04	Cream	43.0	5.08
Cream	22.0	6.95	Cream	44.0	4.99
Cream	23.0	6.86	Cream	45.0	4.90
Cream	24.0	6.77	Cream	46.0	4.81
Cream	25.0	6.68	Cream	47.0	4.72
Cream	26.0	6.59	Cream	48.0	4.63
Cream	27.0	6.50	Cream	49.0	4.54
Cream	28.0	6.42	Cream	50.0	4.45
Cream	29.0	6.33			

Note: Other products are purchased on known composition.

CASE I.—*Milk, Cream, Condensed Skim.*

Several different combinations of ingredients are used in ice cream, the most common one being composed of milk (or skim milk), cream (or butter) and condensed skim. This combination will be treated first and called case I.

Let M = No. lbs. of mix
a = %† fat in milk
b = % fat in cream or butter
c = % ‡M.S.N.F. (Milk-solids-not-fat) in condensed skim
d = % M.S.N.F. in milk or skim milk
e = % M.S.N.F. in cream

* Taken in part from Farrington and Woll.

† Thus if 4% milk is used, $a = 4$

‡ Thus if 32% condensed skim is used, $c = 32$

The sugar, gelatin and water make up 14.5%+0.5%+3.0% or 18% of the mix.

Hence, 100%−18%=82% (or .82) of the mix consists of dairy products; namely, *milk, skim milk*, or *butter and condensed skim*.

Now let X = No. lbs. of *a* % milk required
Y = No. lbs. of *b* % cream required
Z = No. lbs. of *c* % condensed skim required.

Then the formulas are as follows:

$$Z=\frac{10(b-a)+10.5(d-e)-.82(bd-ae)}{c(b-a)-(bd-ae)}\times M.$$

$$Y=\frac{(10.5-.82a)M+aZ}{b-a}.$$

$$X=\frac{10.5M-bY}{a}.$$

It should be remembered that bd means b times d, $bd-ae$ means to subtract a times e from b times d, etc. $10(b-a)$ means to subtract a from b and multiply the difference by 10, etc.

Next, an example is worked out showing how to substitute in the formulas, and then a shorter solution of the same example is given. Afterward the corresponding rules are stated, pages 64 to 66.

Example 1. **(Milk, Cream and Condensed Skim)**

Using standard mix with ingredients having the following composition, to make 1000 lbs. of mix:

Milk, 4% fat
Cream, 35% fat
Condensed skim, 32% M.S.N.F.

Thus:

$a=4$
$b=35$
$c=32$
From Table $\begin{cases} d=8.8^{*} \\ e=5.8 \end{cases}$

* Note that d and e are taken to the nearest 10th only, because using the 100ths will never make more than a fraction of a pound difference in the value of Z for a 1000 lb. mix, and furthermore, these per cents are never known exactly for any given composite milk.

Then substituting these numbers in the formula for Z.

$$Z=\frac{10(35-4)+10.5(8.8-5.8)-.82(35\times 8.8-4\times 5.8)}{32(35-4)-(35\times 8.8-4\times 5.8)}\times 1000$$

$$=\frac{10\times 31+10.5\times 3.0-.82(308.0-23.2)}{32\times 31-284.8}\times 1000$$

$$=\frac{310+31.5-233.5}{992-284.8}\times 1000=\frac{108.0}{707.2}\times 1000$$

$=152.7$ lbs. condensed skim.

Now substituting in the formula for Y, we have

$$Y=\frac{(10.5-.82\times 4)\times 1000+4\times 152.7}{35-4}$$

$$=\frac{7220+610.8}{31}=\frac{7830.8}{31}=252.6 \text{ lbs. of cream.}$$

To find X, we have

$$X=\frac{10.5\times 1000-35\times 252.6}{4}$$

$$=\frac{10500-8841}{4}=414.8 \text{ lbs. of milk.}$$

Preliminary check:

Total dairy products
$=.82\times 1000$ lbs. $=820$ lbs.

152.7
252.6
414.8
820.1 check.

The complete "check" follows:*

10.5% of 1000 lbs. = 105 lbs. of fat required.
In this case there are only two sources of fat, namely, milk and cream.
4% of 414.8 lbs. = 16.59 lbs. fat from milk.
35% of 252.6 lbs. = 88.41 lbs. fat from cream.
105.00 lbs. fat altogether which "checks" with the fat required.

The M.S.N.F. = 10% of 1000 lbs. = 100 lbs. required. There are *three* sources of M.S.N.F. in this case, namely: milk containing 8.8%, cream containing 5.8%, and condensed skim containing 32%.

8.8% of 414.8 = 36.50 lbs. = M.S.N.F. from milk.
5.8% of 252.6 = 14.65 lbs. = M.S.N.F. from cream.
32% of 152.7 = 48.86 lbs. = M.S.N.F. from condensed skim.
100.01 lbs. "Checking" the M.S.N.F.

* It must be noted that the preliminary check given above is not a perfect "check" since the amounts obtained for all the different ingredients might be wrong and yet they might total 820 lbs. Hence the more complete check should always be used.

This solution may seem long, because the substitutions have been made at length in the formulas and all the steps in the process shown. After working out a few mixes in this way and being sure of the meaning of the formulas, the shorter form given below should be used:

The formulas or rules should be kept where they may be referred to quickly and the given numbers written down as here shown.

```
                                                                   35
                                                                  ×8.8
                                                                 ------
                 To find Z:                                        280
                                                                  280            5.8
                   M = 1000                                      ------      ae = ×4
                   a = 4  }                                       308.0          ------
                   b = 35 }  31 = b − a                          −23.2 ............23.2
                   c = 32                        31              ------
                   d = 8.8 }                    ×10              284.8 = bd − ae
                   e = 5.8 } 3.0 = d − e        ----              ×.82
 c(b−a) = 31       ------------                  310              ------
        ×32        10.5×3 = 31.5 .............+31.5              22784 }
        -----                                   ------              570 } *
        992.0                                   341.5             ------ }
 bd − ae = 284.8                                −233.5 ............ 233.54 }
        ------                                  ------
        707.2 ..................................707.2)108.0(.1527 lbs.
        1000×.1527 = 152.7 lbs. condensed skim or Z.

 To find Y:
          .82
          ×4
         -----           10.5
 .82a = 3.28 ............−3.28
                         -------
                          7.22×1000 = 7220.0
                 aZ =     4×152.7   =  610.8
                                      ----------
                             31)7830.8(252.6 lbs. cream or Y.
 To find X:
          10.5×1000 = 10500
          35×252.6  =  8841
                       -------
                   4)1659(414.8 lbs. milk, or X.
```

HINT: Study the short solution given above in connection with the rules.

Rule 1. For finding Z, the amount of condensed skim.

Step 1. Subtract the per cent of fat in the milk from the per cent of fat in the cream: $35-4=31$.

Step 2. Multiply the result of step 1 by the per cent of M.S.N.F. in the mix: $10\times 31=310.0$.

Step 3. Subtract the per cent of M.S.N.F. in the cream from the per cent of M.S.N.F. in the milk: $8.8-5.8=3.0$.

* Contracted multiplication.

Step 4. Multiply the result of step 3 by the per cent of fat in the mix: 10.5×3.0=*31.5.*

Step 5. Add the results of steps 2 and 4: 310+31.5=*341.5.*

Step 6. Multiply the per cent of M.S.N.F. in the milk by the per cent of fat in the cream: 35.0×8.8=*308.0.*

Step 7. Multiply the per cent of M.S.N.F. in the cream by the per cent of fat in the milk: 4.0×5.8=*23.2.*

Step 8. Subtract the result of step 7 from the result of step 6: 308.0−23.2=*284.8.*

Step 9. Multiply the result of step 8 by the per cent, expressed as a decimal, of dairy products in the mix (for standard used): .82×284.8=*233.5.*

Step 10. Subtract the result of step 9 from the result of step 5: 341.5−233.5=*108.0* which is the numerator of the formula.

Step 11. Multiply the result of step 1 by the per cent of M.S.N.F. in the condensed skim: 32×31= *992.0.*

Step 12. Subtract the result of step 8 from the result of step 11: 992.0−284.8=*707.2.*

Step 13. Divide the result of step 10 by the result of step 12: 108.0÷707.2=*.1527.*

Step 14. Multiply the result of step 13 by the number of pounds in the mix: 1000×.1527=*152.7* lbs. of condensed skim required.

Rule 2. For finding Y, the amount of cream required.

Step 15. Multiply the per cent of fat in the milk by the per cent expressed decimally of dairy products in the mix: .82×4.0=*3.28.*

Step 16. Subtract the result of step 15 from the per cent of fat in the mix: 10.5−3.28=*7.22.*

Step 17. Multiply the result of step 16 by the number of pounds in the mix: 1000×7.22=*7220.*

Step 18. Multiply the result of step 14 (the amount of condensed skim required) by the per cent of fat in the milk: 4.0×152.7=*610.8.*

Step 19. Add the results of steps 18 and 17: 7220.0+610.8=*7830.8.*

Step 20. Divide the result of step 19 by the result of step 1 (Rule 1) 7830.8÷31=*252.6* lbs. of cream required.

Rule 3. For finding X, the amount of milk required.

Step 21. Multiply the per cent of fat in the mix by the number of pounds in the mix: 1000×10.5=*10500.*

Step 22. Multiply the result of step 20 (the amount of cream required) by the per cent of fat in the cream: $35.0 \times 252.6 = 8841.0$.

Step 23. Subtract the result of step 22 from the result of step 21: $10500 - 8841 = 1659$.

Step 24. Divide the result of step 23 by the per cent of fat in the milk: $1659 \div 4.0 = 414.8$ lbs. of milk required.

CHECKS: *Check the results in the manner shown on page 63,* which is, briefly, as follows:

Find the amount of fat and M.S.N.F. required for the mix, according to standard used. Then find the amount of fat in the results obtained for all three ingredients, condensed skim, cream and milk. These amounts should agree closely with those required for the total mix.

CASE II. *Milk, cream, condensed product:*

This is the most general case treated. Besides the ingredients considered in Case I, it involves the use of plain condensed whole milk or sweetened condensed whole milk or sweetened condensed skim. The letters a, b, d, and e are used with the same significance as they have in Case I.

But since there may now be another source of fat (the condensed whole milk) and another source of sugar (the sweetened condensed, either whole or skim) the formulas must provide for these also. Hence,

Let h = per cent of fat in the plain condensed or sweetened condensed whole milk, if either of these be used. Otherwise, $h = 0$.

Let k = 100 per cent less the per cent of sugar in the sweetened condensed whole or skim, as the case may be. Thus if the sweetened condensed contains 42% sugar, $k = 100\% - 42\%$ or .58.

As in Case I.

X = No. lbs. of a% milk required,
Y = No. lbs. of b% cream or butter required,
Z = No. lbs. of c% condensed product required.

THE GENERAL FORMULAS FOLLOW:

$$Z = \frac{10(b-a) + 10.5(d-e) - .82(bd-ae)}{c(b-a) + h(d-e) - k(bd-ae)} \times M.$$

$$Y = \frac{(10.5 - .82a)M + akZ - hZ}{b-a}.$$

$$X = \frac{10.5M - bY - hZ}{a}.$$

Example 2. **(Sweetened Condensed Skim)**

Using standard mix with ingredients having the following composition and computing a 1000 lb. mix:

Milk, 4%
Cream, 40%
Sweetened condensed skim milk, 32% M.S.N.F.
(Cane) sugar, 42%.

Thus $M=1000$

$a=4$
$b=40$
$c=32$
$d=8.6$
$e=5.4$
$h=0$ (since the fat in the skim milk is practically Zero)
$k=.58$ ($=100\%-42\%$)

Computing Z first:

$$Z=\frac{10(40-4)+10.5(8.6-5.4)-.82(40\times 8.6-4\times 5.4)}{32(40-4)+0(8.6-5.4)-.58(40\times 8.6-4\times 5.4)}\times 1000$$

$$=\frac{10\times 36+10.5\times 3.2-.82\times 322.4}{32\times 36+0-.58\times 322.4}\times 1000$$

$$=\frac{360+33.6-264.4}{1152-187}\times 1000=\frac{129200}{965}=133.9 \text{ lbs. of sweetened condensed skim.}$$

$$Y=\frac{(10.5-.82\times 4)\times 1000+4\times .58\times 133.9-0\times 133.9}{40-4}$$

$$=\frac{7220+310.6-0}{36}=\frac{7530.6}{36}=209.2 \text{ lbs. cream.}$$

$$X=\frac{10500-8368-0}{4}=\frac{2132}{4}=533.0 \text{ lbs. milk.}$$

Condensed computation of Example 2. (*Note:* The steps are performed in order according to the formulas.)

Solving for Z first:

```
          8.6        8.6
         -5.4        ×40          5.4
 40      ----       -----         ×4
 -4       3.2       344.0        ----
----     ×10½       -21.6.........21.6
 36      ----       -----
 10       320       322.4=bd-ae
----       16        ×.82
360      ----       -----
 33.6.....33.6      25792
----                  645
393.6               -----
264.4...............264.37
----
129.2 Numerator of formula.
```

Finding the value of the denominator:

```
  36   Since any number times zero is
 ×32   zero the second term of the
-----  denominator is Zero
  72   The third term is .58 times the
 108   bd-ae already found, i. e.,
-----  .58×322.4=187.0.
1152
  1152-187.0=965, the Denominator.
```

Dividing the numerator by the denominator,

129.2÷965=.1339

1000×.1339=133.9 lbs. sweetened condensed skim.

Solving for Y:

```
 .82
 ×4
 ----      10.5
 3.28 ....−3.28
           -----
           7.22 × 1000 = 7220
 .58
 ×4
 ----      133.9
 2.32 .....×2.32
           -----
      40   310.6
      −4   7220.0
      --   ------
      36 ) 7539.6 (209.2lbs. of cream.
```

Check as before.

Solving for X:

```
 10.5
 ×1000                     209.2
 -------                     ×40
 10500.0                   -----
  8368.0 ..................8368.0
 -------
 4)2132.0
  533 lbs. of milk.        133.9
                              ×0
                           -----
                            0000
```

Preliminary "check":

533.0 lbs. milk
209.2 lbs. cream
133.9 lbs. sweetened condensed skim
35.0 lbs. gelatin and water
88.8 lbs. sugar to be added (since the total amount of sugar is 145 lbs. and the 133.9 lbs. of condensed skim contains .42×133.9 or 56.2 lbs)
999.9 145−56.2=88.8.

The caution given in the foot note on page 63 about checking the results should be heeded here also.

The complete check follows:

As in the "check" for Example 1,

10.5% of 1000 lbs.=105 lbs. of fat required.

The sources of fat are the 4% milk and 40% cream.

4% of 533.0 lbs. = 21.32 lbs. fat from milk,
40% of 209.2 lbs. = 83.68 lbs. fat from cream.
105.00 lbs. Total fat, which checks.

10% of 1000=100 lbs. of M.S.N.F. required.

The sources of M.S.N.F. are 8.6% from milk, 5.4% from cream, and 32% from sweetened condensed skim.

8.6% of 533.0 lbs. = 45.84 lbs. M.S.N.F. in milk.
5.4% of 209.2 lbs. = 11.30 lbs. M.S.N.F. in cream.
32% of 133.9 lbs. = 42.75 lbs. M.S.N.F. in sweetened condensed.
99.89 lbs. Total M.S.N.F. which checks the required 100 lbs. nearly enough.

(It is probably superfluous to state that carrying the results out further would give a still closer check.)

Example 3. **(Plain Condensed Whole Milk)**

To make 1000 lbs. of standard mix with the following ingredients:

Milk, 3.4% fat
Cream, 38% fat
Plain condensed whole milk {8% fat, 23% M.S.N.F.}

This example comes under Case II, but since the plain condensed whole milk contains no sugar, $k = 100\% - 0\% = 1$, so that the last term in the denominator of the formula is just $1(bd - ae) = bd - ae$.

Henceforth only the short form of the solutions will be given.

Finding Z first:

$a = 3.4$
$b = 38$
$c = 23$
From Table { $d = 8.7$ (since 3.4 milk is nearly 3.5); $e = 5.5$ }
$h = 8$
$k = 1$

$b - a =$ { 38.0, 3.4 }
34.6
×10
346
+33.6 .... 33.6
379.6
−255.8 ....
Num. = 123.8

8.7
5.5
3.2 $= d - e$
10½
320
16
33.6

Dividing Numerator by Denominator

123.8 ÷ 509.5 = .2430
1000 × .243 = 243 lbs. condensed milk = Z.

$b - a = 34.6$
$c = \times 23$
1038
692
$c(b-a) = 795.8$

$d - e = 3.2$
$h = \times 8$

$h(d-e) = 25.6$ } add
$c(b-a) = 795.8$ }
821.4
$k(bd - ae) = 311.9$ since $k = 1$
509.5 Denom.

$bd =$ { 38; ×8.7; 266; 304; 330.6 }

$ae =$ { 3.4; ×5.5; 170; 170; 18.70 }

330.6
−18.7
311.9 $= bd - ae$
×.82
24952
624
........255.76

Finding Y:

```
    a=3.4                           {  243
     ×.82                           { ×3.4
     ----                     akZ = {  ----
      272                           {  972
       7                            {  729
     ----                           {  ----
     2.79 } Subtract                { 826.2
    10.50 }
     ----                           {  243
     7.71                      hZ = {   ×8
    ×1000                           {  ----
  7710.00 } Add                     { 1944
   826.00 }
akZ = ----
  8536.0 } Subtract
  1944.0 }
hZ = ----
  6592.0 = Num.

  b−a=34.6=Denom.
  6592÷34.6=190.52 lbs. cream
```

Finding X:

```
   10.5×1000=10500
  190.52
    ×38
  ------
  57156
  15242
  ------
  7239.8

  10500.0
   7239.8
  -------
   3260.2
hZ= 1944
  -------
3.4) 1316.2(387.1 lbs. milk.
```

Preliminary check:

.82×1000=820 lbs. dairy products

```
      243.0
      190.5
      387.1
     ------
      820.6 which checks near-
            ly enough.
```

Complete check:

3.4% of 387.1 lbs. = 13.16 lbs. fat from milk.
38% of 190.5 lbs. = 72.40 lbs. fat from cream.
8% of 243.0 lbs. = 19.44 lbs fat from condensed milk
105.00 check.

10% of 1000 lbs. = 100 lbs. M.S.N.F. required.

Three sources, 8.7% in milk, 5.5% in cream, and 23% in condensed whole milk.

8.7% of 387.1 lbs. = 33.67 lbs. from milk,
5.5% of 190.5 lbs. = 10.48 lbs. from cream,
23% of 243.0 lbs. = 55.89 lbs. from condensed whole.
100.04 check.

The most general problem under Case II, arises when sweetened condensed whole milk is used. Then both h and k have special values in the formulas.

Example 4. **(Sweetened Condensed Whole Milk)**

To make 1000 lbs. of standard mix using:

Sweetened condensed whole { 8% fat, 23% M.S.N.F., 40% sugar }

Milk 4%
Cream 35%

$a=4$ $h=8.0$
$b=35$ $k=100\%-40\%=.60$
$c=23$
From Table { $d=8.8$, $e=5.8$ }

Substituting in the general formulas of Case II, page 66:

$$Z=\frac{10(35-4)+10.5(8.8-5.8)-.82(35\times 88-4\times 5.8)}{23(35-4)+8.0(8.8-5.8)-.60(35\times 8.8-4\times 5.8)}\times 1000.$$

Note: The numerator for Z is the same as in Example 1, page 63, and, hence, equals 108.0.

For the denominator:

```
b−a = { 35
        −4
       ----
        31        d−e = { 8.8        3.5
        23                5.8        8.8
       ----              -----      -----
        93                3.0        280
        62               ×8.0        280         5.8
       ----              -----      -----        ×4
       713 ............  24.0       308.0       ----
                         713.0       23.2 ........ 23.2
                         -----      -----
                         737.0      284.8
                                     .60
                                    -----
                         179.9 ..... 170.88
                         -----
                         566.1 = Denom.
```

108.0 ÷ 566.1 = .1908

1000 × .1908 = 190.8 lbs. sweetened condensed whole.

$$Y=\frac{(10.5-.82\times 4)\times 1000+4\times .60\times 190.8-8\times 190.8}{35-4}.$$

```
 .82
 ×4                        190.8
-----                       ×.60
 3.28                      ------
10.5                       114.480
-----                        ×4
 7.22×1000 = 7220.0        ------
             +457.9 ...... 457.92              190.8
             ------                              ×8
             7677.9                            ------
            −1526.4 ......................... 1526.4
             ------
         31)6151.5(198.4 lbs. cream.
```

$$X=\frac{10.5\times 1000-35\times 198.4-8\times 190.8}{4}.$$

```
               189.4
                ×35
 10.5          -----
 ×1000          5952
------          992
10500          -----
 6944 ........ 6944           190.8
------                          ×8
 3556                         -----
1526.4 ...................... 1526.4
------
4)2029.6
  ------
  507.4 lbs. milk
```

Check: Here the sweet condensed whole includes sugar equal to 40% of 190.8 or 76.32. Hence, only 190.8−76.3 or 114.5 lbs. of it is strictly "dairy products".

114.5+198.4+507.4=820.3 lbs. which checks the 820 lbs. dairy products required.

4% of 507.4=	20.30 lbs. from milk,
8% of 190.8=	15.25 lbs. from sweet condensed whole,
35% of 198.4=	69.44 lbs. cream.
	105.00 which checks fat needed.
8.8% of 507.4=	44.65 lbs. from milk,
23% of 190.8=	43.88 lbs. from condensed whole,
5.8% of 198.4=	11.51 lbs. from cream.
	100.04 lbs. which checks M.S.N.F. needed.

Miscellaneous.

Under this head are classed those problems in which some of the ordinary ingredients are missing or given amounts of some of them are used.

Example 5. (**Cream and Condensed Skim only**)

To make 1000 lbs. of mix using only cream and condensed skim (no whole milk) so that $a=0$ and $d=0$.

$a=0$
$b=40$
$c=30$
$d=0$
$e=5.35$

This problem comes under Case I, since h and k do not appear; and the formulas for Z and Y become:

$$Z=\frac{10b-10.5e}{cb}\times M; \qquad Y=\frac{10.5M}{b}.$$

Finding Z:

```
                6.2        40
               ×10½        30
40            ─────      ─────
×10            620       1200 Denom.
─────           31
400           ─────
65.1............65.1

334.9÷1200=.2791
1000×.2791=279.1 lbs. condensed
                        skim
```

Finding Y:

```
10.5×1000=10500
10500÷40  =262.5 lbs. cream,
           279.1 lbs. skim.
           ─────
           541.6 lbs. dairy products.
```

But since only 18% of the mix is accounted for in the sugar, gelatin and water, we must still account for 82% of it or 820 lbs. Hence, we must add water enough to make this amount.

```
820
541.6
─────
278.4 lbs. water to be added.
```

Example 5a. **(Maximum Amount of Cream)**

The maximum amount of cream will be used if cream only is used as a source of fat and this problem is solved in example 5 above. Of course, if the cream is very rich, say 30 to 40%, water will have to be added. Example 5 shows how to compute the amount of cream and condensed skim and how to find the amount of water to be added.

Example 6. **(Condensing Problem)**

No condensed product is used, but the mix is first made up larger than it should be finally and then condensed. Since no condensed product enters into this mix, there is no formula for Z. The formulas for X and Y follow:

$$X=\frac{10b-10.5e}{bd-ae}\times M \qquad Y=\frac{10.5d-10a}{bd-ae}\times M$$

where the letters have the usual meanings, as explained on page 61.

To make 1000 lbs. of standard mix using 4% milk and 40% cream and condensing down, thus:

$a=4$
$b=40$
$c=0$ (does not appear)

$d=8.8$ } From table
$e=5.35$ }

Substituting in the formulas for X and Y:

$$X=\frac{10\times40-10.5\times5.35}{40\times8.8-4\times5.35}\times1000=\frac{400-56.17}{352-21.4}\times1000$$

$$=\frac{343.83}{330.6}\times1000=1040 \text{ lbs. milk}$$

$$Y=\frac{10.5\times8.8-10\times4}{330.6}\times1000=\frac{92.4-40}{330.6}\times1000$$

$$=\frac{52.4}{330.6}\times1000=158.5 \text{ lbs. cream.}$$

1040 lbs. milk
158.5 lbs. cream
180 lbs. sugar, gelatin and water (=18% of 1000 lbs.)

1378.5 Total to be condensed down to 1000 lbs.

Example 7. **(Cream, Skim and Condensed Skim)**

To make 1000 lbs. standard mix with following ingredients.

30% cream,
Plain skim, 8.9% M.S.N.F.
32% condensed skim

The formulas of Case I, page 62, are used, where the skim milk takes the place of the whole milk.

Hence,

$$a=0$$
$$b=30$$
$$c=32$$
$$\left.\begin{matrix} d=8.9 \\ e=6.2 \end{matrix}\right\} \; 2.7=d-e.$$

The formulas become:

$$Z=\frac{10b+10.5(d-e)-.82\,bd}{b(c-d)}\times M.$$

$$Y=\frac{10.5M}{b}. \qquad X=.82M-(Y+Z).$$

Substituting in the formulas,

$$Z=\frac{10\times30+10.5\times2.7-.82\times30\times89}{30(32-8.9)}\times 1000.$$

```
30            10.5         .82       |        32.0
10             2.7         ×30       |         8.9
----          -----       -----      |        -----
300            735        24.60      |        23.1
               210          8.9      |        ×30
                                     |        -----
 28.35.........28.35       2214      |  Denom. 693.0)109.41(.1579
                           1968      |
----                      -----      |  1000×.1579=157.9 or 158 lbs. con-
328.35                               |    densed skim.
218.94....................218.94     |
----                                 |
Num. 109.41                          |
```

Finding Y:

$$Y=\frac{10.5\times1000}{30}=\frac{10500}{30}=350\text{ lbs. cream.}$$

```
  350              .82×1000=820
 +158               X=820−508=312 lbs. skim.
 ----
  508 lbs.
```

Check: 30% of 350 lbs. = 105 lbs. fat from cream, which checks, as the cream is the only source of fat.

M.S.N.F. {
6.2% of 350 lbs. = 21.70 from cream.
8.9% of 312 lbs. = 27.77 from skim,
32% of 158 lbs. = 50.56 from condensed skim.
}

100.03 lbs. total which checks 100 lbs. required.

Example 8. **(Left Overs:** One may have a certain amount of cream or milk or both left over which must be used in the mix.)

To make 1000 lbs. standard mix using:

60 lbs. left over 35% cream,
50 lbs. left over 20% cream,
150 lbs. left over 4% milk,
and enough 3.8% milk and 40% cream and 30% condensed skim to make the mix. It is necessary to find first how much fat and M.S.N.F. are contained *in the milk and cream left over which must go into the mix.*

35% of 60 lbs. = 21.0 lbs. of fat from cream,
20% of 50 lbs. = 10.0 lbs. of fat from cream,
3.8% of 150 lbs. = 5.7 lbs. of fat from milk,
36.7 lbs. fat accounted for.

10.5% of 1000 = 105.0 lbs. fat required altogether.
105.0 lbs. − 36.7 = 68.3 lbs. still needed.
68.3 ÷ 1000 = .0683 or 6.83%; that is, the fat still needed equals 6.83% of the 1000 lbs. mix.
Hence, the 6.8% (close enough) takes the place of the old 10.5 in the formulas of Case I.

Similarly for M.S.N.F.

5.8% of 60 lbs. = 3.48 M.S.N.F. from cream.
7.1% of 50 lbs. = 3.55 M.S.N.F. from cream.
8.8% of 150 lbs. = 13.20 M.S.N.F. from milk.
20.23 = M.S.N.F. accounted for.

10% of 1000 lbs. = 100 lbs. M.S.N.F. required altogether.
100 − 20.23 = 79.77 or 80 lbs. M.S.N.F. still needed.
80 ÷ 1000 = .080 or 8%.

Hence the 8.0 takes the place of the old 10 in the formulas.

Now, as usual with the standard mix, 82% of it consists of dairy products amounting to 820 lbs. The 60+50+150 to be used amounts to 260 lbs.

Hence, 820 lbs. − 260 lbs. or 560 lbs. equals amount still needed.
560 ÷ 1000 = .56 which takes the place of the old .82 in the formulas.

Hence, the formula for Z, Case I, page 62, is:

$$Z=\frac{8.0(b-a)+6.3(d-e)-.56(bd-ae)}{c(b-a)-(bd-ae)}\times M$$

$$Y=\frac{(6.8-.56a)M+aZ}{b-a}$$

$$X=\frac{6.8M-bY}{a}$$

The problem now is to find how much of the 3.8% milk, 40% cream and 30% condensed skim must be used to make up the 560 lbs. still needed, which is done by substituting the proper values of a, b, etc., in the formulas as written above.

In this particular example:

$a = 3.8$
$b = 40$
$c = 30$
From Table $\begin{cases} d = 8.8 \\ e = 5.35 \end{cases}$

40.0
3.8
———
36.2
×8
———
289.6
+23.5 .... 23.45
———
313.1
185.75 ....
———
127.35 = Num.

$d-e = \begin{cases} 8.80 \\ 5.35 \end{cases}$
———
3.45
×6.8
———
2070
276
———
23.45

$bd = \begin{cases} 8.8 \\ \times 40 \end{cases}$
———
352.0

$ae = \begin{cases} 5.35 \\ \times 3.8 \end{cases}$
———
1605
428
———
20.33

352.0
−20.3
———
331.7 $= bd - ae$
×.56
———
16585
1990
———
185.75

Finding Denom.:

$c(b-a) = \begin{cases} 36.2 \\ \times 30 \end{cases}$
———
1086.0
$bd-ae$ 331.7
———
754.3 = Denom.
127.35 ÷ 754.3 = 168.8 lbs. condensed skim required.

Finding Y:

.56 168.8
3.8 3.8
——— ———
168 5064
45 1350
——— ———
2.13 641.4

6.80
2.13
———
4.67 × 1000 = 4670.0
641.4
———
$b-a = 36.2$)5311.4(146.7 lbs. cream.

Finding X:

6.8 × 1000 = 6800.0
bY = 40 × 146.7 = 5868.0
3.8)932.0(245.3 lbs. milk

Check:
245.3
146.7
168.8
———
560.8 lbs. which checks the 560 lbs. needed (near enough.)

Fat $\begin{cases} 40\% \text{ of } 146.7 = 58.68 \text{ from cream,} \\ 3.8\% \text{ of } 245.3 = 9.32 \text{ from milk,} \end{cases}$
68.00 Total,
which checks the 68.3 lbs. needed:

30% of 168.8 = 50.65 lbs. from skim,
5.35% of 146.7 = 7.85 lbs. from cream,
8.8% of 245.3 = 21.60 lbs. from milk,
———
80.09 lbs. Total M.S.N.F.,
which checks the 80 lbs. needed.

Example 9. **(Having to use a given amount of Sweet Butter and Cream)**

To make 1000 lbs. of standard mix, using
100 lbs. sweet butter, 83% fat;
50 lbs. of 25% cream with enough 40% cream (no whole milk) and 32% condensed skim to make the mix.

It is necessary first to find how much fat and M.S.N.F. the given ingredients contain, then to find how much must be furnished by the unknown 40% cream and condensed skim.

83% of 100 lbs. = 83 lbs. fat in the butter.
25% of 50 lbs. = 12.5 lbs. fat in the 25% cream.

95.5 lbs. fat in both.

10.5% of 1000 lbs. = 105 lbs. in the mix.
105 − 95.5 = 9.5 lbs. fat still needed.

Since this 9.5 lbs. of fat is to come from 40% cream, 9.5 ÷ .40 = 23.75 lbs. of 40% cream needed.

100 + 50 + 23.75 = 173.8 lbs. total cream and butter.

Now, in order to use the formulas, and thus avoid "cutting and trying," it is necessary to find what this cream and butter test, both in fat and M.S.N.F.

105 ÷ 173.8 = 60.4% fat = b in the formulas.

The 25% cream tests 6.68% or 6.7% M.S.N.F. by the table and the 40% cream tests 5.35%

6.7% of 50 lbs. = 3.35 lbs. M.S.N.F. in 25% cream.
5.35% of 23.75 = 1.27 lbs. M.S.N.F. in 40% cream.

Total = 4.62 lbs. M.S.N.F. in cream and butter.

Now since no milk is used, a and e are Zero (= 0).

Hence:

$a = 0$ $d = 0$
$b = 60.4$ $e = 2.65$
$c = 32$

Y, the cream and butter is already known, namely 173.8 lbs. and the formula for Z becomes:

$$Z = \frac{10b - 10.5e}{cb} \times M.$$

Substituting the new values of *b* and *e* in the formulas for Z,

$$Z = \frac{10 \times 60.4 - 10.5 \times 2.65}{32 \times 60.4} \times 1000.$$

```
                   26.5
                   ×10½
 60.4              ----            60.4
  ×10              26.5              32
 -----              1.32           ----
 604.0             -----           1208
  27.82........... 27.82           1812
 -----                             ----
Num.=576.18                        1932.8=Denom.
```

576.18÷1932.8=.298

1000×.298=298 lbs. condensed skim.

Check as usual.

Special Problem in which given amounts of two or three ingredients are used, and which can be solved without either the algebraic formulas or "cutting and trying."

Example 10. **(Given amounts of two or more ingredients)**

To make 1000 lbs. of standard mix with the following given amounts of milk and cream.

200 lbs. of 4% milk
100 lbs. of 3% milk
100 lbs. of 35% cream and enough 40% cream and 30% skim to make the mix.

Solution:

4% of 200 lbs. = 8 lbs. fat,
3% of 100 lbs. = 3 lbs. fat,
35% of 100 lbs. = 35 lbs. fat.

46 lbs. fat in given ingredients.

105 lbs. − 46 lbs. = 69 lbs. fat still to come from 40% cream.
69 ÷ .40 = 172.5 lbs. of 40% cream needed.

8.8% of 200 lbs. = 17.6 lbs. from 4% milk,
8.6% of 100 lbs. = 8.6 lbs. from 3% milk,
5.8% of 100 lbs. = 5.8 lbs. from 35% cream,
5.35% of 172.5 = 9.2 lbs. from 40% cream.

41.2 lbs. provided for.

100 lbs. − 41.2 = 58.8 lbs. M.S.N.F. still to come from condensed skim testing 30%. 58.8 lbs. ÷ .30 = 196 lbs. condensed skim.

Hence 172.5 lbs. of 40% cream and 196 lbs. of condensed skim are needed in addition to the given ingredients.

Other Standards: The formulas and rules given above can be applied to other standard mixes than the one used thus far in this bulletin.

Example: To make 1000 lbs. (or any other amount) of mix which shall contain the following:

12% fat.
11% M.S.N.F.
14% sugar.
0.5% gelatin with no water.

14% sugar + 0.5 gelatin = 14.5%.

Hence, the dairy products must constitute 100% − 14.5% = 85.5% or .855 of the mix and the numbers 12, 11, and .855 simply take the place of 10.5, 10 and .82 in the formulas of Case I and Case II. Thus the formulas of Case I become:

$$Z=\frac{11(b-a)+12(d-e)-.855(bd-ae)}{c(b-a)-(bd-ae)}\times M,$$

where a, b, c, etc., have the same meaning as in previous examples, and M is 1000 or 750 or any other amount of mix desired.

$$Y=\frac{(12-.85a)M+aZ.}{b-a}$$

$$X=\frac{(12M-bY).}{a}$$

The rules on pages 64 to 66 apply here also.

Thus in step 2, the result of step 1 is multiplied by 11 under this standard, instead of 10: and in step 9, the result of step 8 is multiplied by .855 instead of .82, and so on.

7m-5,'25